

Derivation of Datum Transformation Parameters for Dubai Emirate

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Key words: Datum, Seven Parameters, Dubai

SUMMARY

A Detail study about the transformation of parameters between two spheroid CLARK1880 and WGS84 in Dubai Emirates was carried out. Due to the non-homogeneity of the control on the Clark1880, made this study interesting even in a small area. The huge common control on the both the datum was used. Originally there were lot of control available on the Clark1880 with spirit level height and 70% of the same control points were observed with GPS on WGS84 spheroid. For the computation of transformation of parameters purpose, the area was classified into two parts one is Mainland and another is Hatta, which are 50 km apart.

Using the geoid solution and ellipsoidal solution, five sets of transformation parameters between the two systems were estimated for each solution. Both Bursa-Wolf and Molodensky-Badekas models were considered in this study. Particular attention was given to the conversion of orthometric heights to their corresponding ellipsoidal heights in the Clark1880 ellipsoid, through using Abridged Molodensky formulas and using the Dubai precise Geoid model.

The role of the geoid in estimating the transformation parameters is well defined. The optimal datum transformation parameters between the WGS84 datum and Clark1880 were determined, which is based on 2966 common points for the main-land and 88 common points in Hatta region with standard deviation of 0.15 cm in planimetry for the main-land and 0.13m for the Hatta region. The two sets of different seven optimal transformation parameters for the mainland and Hatta region were computed. A total of 3744 and 91 control points were tested for the mainland and Hatta region respectively. A Seven-parameter transformation on both models was found to be superior comparing to all other sets of parameters.

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1. INTRODUCTION

Survey control in Dubai Emirate dated back to 1927- 1931, when the old Trucial Coast Countries third order Geodetic Control was established. Initially only *FOUR* control stations from SW corner of this series were used for Survey Control networking. During 1978-1980 for precise mapping projects a major densification of control networks was carried out by major/minor triangulation and further densification by traverses. The Hatta region (which is about 50 km away from main land) was connected to main land by Traverse and Trilateration. The control networks started from SW and when closing the networks, there were no known control available in the South East corner. Only the bearing check was established. The above Trucial Coast points have their coordinates based on the Clark 1880 Ellipsoid (modified Neharwan datum) and vertical control is based on Port Rashid Datum refer DGN FILE1. The planimetric coordinates computed in UTM Zone 40. During 1991 the First order GPS network is planned and it was composed of 62 monuments (which was already having coordinates on Clark 1880). The network is more dense in the city area where as in the desert the distances between control stations were from 5 to 20 Km. GPS observations were made for these quadrilaterals and polygons. The DUREF-95 campaign determined the geocentric coordinates for 5 control stations in Dubai Emirate in the WGS84 ITRF93. After deriving the ITRF coordinates of 5 stations the old network have been re-adjusted using the ITRF93 derived coordinates. About 6000 stations which were earlier existing on Clark1880 have been re-observed in 1996 - 2000, using Stop & Go GPS technique and tied to WGS84-ITRF system.

About 9000 control stations on Clark 1880 was established through out Dubai Emirate in a span of 20 years. Due to the enormous construction activities about 40% of the stations have been destroyed. About 50% of stations have been connected with spirit level heights. The whole control networks was based on 4 SW corner control points of Trucial coast survey without closing on any other known control at the end, hence it was believed that the control network were not homogeneous. There was an attempt to develop the transformation of parameters during 1997 by Dr Racordo Passini. The network of 62ET (Electronic Traverse) points with coordinates on the clark1880 and WGS 84 were taken to develop the parameters. In one of his notes he mentioned that “ using the above 62 ET points with coordinates on the above two systems, several transformations model have been investigation with non-acceptable results. The tested models were

”Molodenski series (abridge and complete), 7 Parameters, 4 parameters (conformal), 6 Parameters (Affine), 8 Parameters polynomial, 10 Parameters polynomial, Bi –cubic 16 parameters transformation”

Finally he used the Bi –cubic 16 parameters for developing the transformation. He developed a method to estimate the parameters using simple interpolation procedures based on a rectangular grid derived from the residuals at reference(62) points and using the Kriging interpolation strategy.

During the year 1999 when Dubai Municipality switched over to the Dubai Local Transverse Mercator Projection (DLTM) system on WGS84 datum, a decision was taken not to go for transformation of existing Clark1880/UTM coordinate for the various above mention reasons. Instead they observed the existing ground control by GPS on WGS84 Spheroid and converted to DLTM projection. But many instances there were necessity to transform the clark1880 coordinates to DLTM as many old survey information were still on Clark1880. Now an attempt is made to develop a single set of numerical transformation parameters for the main land of Dubai and for Hatta region.

2. MATHEMATICAL MODELS

2.2 Co-ordinates transformations

Three-dimensional co-ordinates could be transformed from Cartesian to curvilinear or vice versa through the knowledge of the parameters of an adopted reference ellipsoid.

2.2.1 The Cartesian co-ordinates transformation

The forward transformation from geodetic co-ordinates (ϕ, λ, h) to Cartesian co-ordinates (X, Y, Z) is given in Heiskanen and Moritz (1967) as:

$$X = (v + h) \cos \phi \cos \lambda \quad (1)$$

$$Y = (v + h) \cos \phi \sin \lambda \quad (2)$$

$$Z = (v(1 - e^2) + h) \sin \phi \quad (3)$$

where the prime vertical radius of curvature (V) is:

$$v = \frac{a}{(1 - e^2 \sin^2 \phi)^{\frac{1}{2}}} \quad (4)$$

with:

a : the semi-major axis of the reference ellipsoid;

e : the first eccentricity of the reference ellipsoid.

2.2.2 The Curvilinear co-ordinates transformation

The non-iterative reverse transformation from Cartesian co-ordinates (X, Y, Z) to geodetic co-ordinates (ϕ, λ, h) is given in Bowring (1985) as:

$$\phi = \arctan \left[\frac{Z + \varepsilon^2 b \sin^3 u}{P - e^2 a \cos^3 u} \right] \quad (5)$$

$$\lambda = \arctan \left[\frac{Y}{X} \right] \quad (6)$$

$$h = P \cos \phi + Z \sin \phi - a \sqrt{1 - e^2 \sin^2 \phi} \quad (7)$$

$$\text{With: } u = \arctan \left[\frac{aZ}{bP} \right], \quad P = \sqrt{X^2 + Y^2}, \quad \varepsilon = \frac{e^2}{1 - e^2}$$

Where,

- u : the parametric latitude;
- b : the semi-minor axis of the reference ellipsoid;
- ε : the second eccentricity of the reference ellipsoid.

2.3 Gravimetric Geoid

As is well known, the geoidal height N , at any point of geographical co-ordinates (ϕ, λ) can be computed from a surface integral derived by Stokes in 1949 as:

$$N(\phi, \lambda) = \frac{R}{4\pi\gamma} \iint \Delta g S(\psi) d\sigma \quad (8)$$

where:

- R : mean radius of the earth;
- γ : mean value of gravity over the earth;
- Δg : free air gravity anomaly;
- $S(\psi)$: original Stokes's Kernel
- $d\sigma$: surface element

In this paper a procedure to obtain the gravimetric geoid is based on the combination of spherical harmonics potential coefficient set with terrestrial gravity data. The final formulae to compute gravimetric geoid heights were given in [Forsberg 2001] as follows:

$$N = N^S + N^C + \Delta N \quad (9)$$

Where,

- N^S : long wavelength component
- N^C : short wavelength component
- ΔN : is the Truncation error

$$N^S = R \sum_{n=2}^L \sum_{m=0}^n [\bar{C}_{nm}^* \cos m\lambda + \bar{S}_{nm}^* \sin m\lambda] \bar{P}_{nm}(\sin \bar{\phi}) \quad (10)$$

$$N^C = \frac{R}{4\pi\gamma} \iint_{\sigma_c} S_L^m(\psi) \Delta g^L d\sigma \quad (11)$$

$$\Delta N = \frac{R}{2\gamma} \sum_{N=2}^L \Delta g_N \bar{Q}_N^m(\psi_o) \quad (12)$$

where;

- $S_L^m(\psi)$: the modified Stokes's kernel

$Q_n^L(\psi_0)$: truncation coefficient associated with the modified Stokes's kernel
 L : the degree of the spheroid
 $\bar{C}_{n,m}^*, \bar{S}_{n,m}^*$: are the fully normalised potential coefficients
 $\bar{\phi}$: geocentric latitude
 λ : geocentric longitude
 $\bar{P}_{nm}(\sin \bar{\phi})$: associated Legendre function
 Δg^L : residual gravity anomaly
 Δg_n : Laplace's surface harmonic of degree n

2.4 Transformation of local geodetic system CLARK1880 to WGS84 system

2.4.1 The Abridged Molodensky Formulas

This datum transformation model as used by the National Imagery and Mapping Agency (NIMA), the former Defence Mapping Agency (DMA) is a five parameter transformation model, which transforms three dimensional curvilinear co-ordinates between two datums. It simply applies the three dimensional geocentric datum shift parameters ($\Delta X, \Delta Y, \Delta Z$) in conjunction with the differences between the semi-major axis (Δa) and flattening (Δf) of the local geodetic system ellipsoid and the WGS84 ellipsoid respectively (WGS84 minus local). From Defence Mapping Agency (1987), the Abridged Molodensky transformation in curvilinear form is given as:

$$\Delta \phi'' = \frac{[-\Delta X \sin \phi \cos \lambda - \Delta Y \sin \phi \sin \lambda + \Delta Z \cos \phi + (a\Delta f + f\Delta a) \sin 2\phi]}{\rho \sin 1''} \quad (13)$$

$$\Delta \lambda'' = \frac{[-\Delta X \sin \lambda + \Delta Y \cos \phi]}{\nu \cos \phi \sin 1''} \quad (14)$$

$$\Delta h_m = \Delta X \cos \phi \cos \lambda + \Delta Y \cos \phi \sin \lambda + \Delta Z \sin \phi + (a\Delta f + f\Delta a) \sin^2 \phi - \Delta a \quad (15)$$

with: $\rho = \frac{a(1-e^2)}{(1-e^2 \sin^2 \phi)^{\frac{3}{2}}}$ (16)

where;

ϕ, λ, h : geodetic co-ordinates of the local geodetic system ellipsoid;
 $\Delta \phi, \Delta \lambda, \Delta h$: corrections to transform local datum co-ordinates to WGS84 ϕ, λ, h ;
 $\Delta X, \Delta Y, \Delta Z$: corrections to transform local datum co-ordinates to WGS84 X, Y, Z ;
 $\Delta a, \Delta f$: (WGS84 minus local) semi-major axis and flattening respectively;
 a : semi-major axis of the local geodetic system ellipsoid;
 f : flattening of the local geodetic system ellipsoid;
 ρ : radius of curvature in the meridian.

2.4.2 The Bursa-Wolf Transformation Model

The Bursa-Wolf (Bursa, 1962; Wolf, 1963) is a seven-parameter model for transforming three-dimensional Cartesian co-ordinates between two datums. This transformation model is more suitable for satellite datums on a global scale (Krakwisky and Thomson, 1974). The transformation involves three geocentric datum shift parameters ($\Delta X, \Delta Y, \Delta Z$), three rotation elements (R_X, R_Y, R_Z) and scale factor ($1 + \Delta L$). The model in its matrix-vector form could be written as (see Burford 1985) :

$$\begin{bmatrix} X_{WGS84} \\ Y_{WGS84} \\ Z_{WGS84} \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} 1 + \Delta L & R_Z & -R_Y \\ -R_Z & 1 + \Delta L & R_X \\ R_Y & -R_X & 1 + \Delta L \end{bmatrix} \begin{bmatrix} X_{CLK} \\ Y_{CLK} \\ Z_{CLK} \end{bmatrix} \quad (17)$$

where;

$X_{WGS84}, Y_{WGS84}, Z_{WGS84}$: are the global datum (WGS84) Cartesian co-ordinates;

$X_{CLK}, Y_{CLK}, Z_{CLK}$: are the local datum (CLARK1880) Cartesian co-ordinates.

2.4.3 The Molodensky-Badekas Transformation Model

The Molodensky-Badekas (Molodensky et al., 1962; Badekas, 1969) is also a seven-parameter model for transforming three-dimensional Cartesian co-ordinates between two datums. This transformation model is more suitable for transformation between terrestrial and satellite datums, (Krakwisky and Thomson, 1974). The transformation also involves three barycentric datum shift parameters (dX, dY, dZ), three rotation elements (R_X, R_Y, R_Z) and scale factor ($1 + \Delta L$). This transformation model is theoretically identical to the Bursa-Wolf model. The model in its matrix-vector form could be written as (see Burford 1985):

$$\begin{bmatrix} X_{WGS84} \\ Y_{WGS84} \\ Z_{WGS84} \end{bmatrix} = \begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix} + \begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} + \begin{bmatrix} 1 + \Delta L & R_Z & -R_Y \\ -R_Z & 1 + \Delta L & R_X \\ R_Y & -R_X & 1 + \Delta L \end{bmatrix} \begin{bmatrix} X_{CLK} - X_m \\ Y_{CLK} - Y_m \\ Z_{CLK} - Z_m \end{bmatrix} \quad (18)$$

with:

$$X_m = \frac{1}{n} \sum_{i=1}^n X_i ; Y_m = \frac{1}{n} \sum_{i=1}^n Y_i ; Z_m = \frac{1}{n} \sum_{i=1}^n Z_i \quad (19)$$

where;

X_i, Y_i, Z_i : are the Cartesian co-ordinates in the local CLARK1880 system;

n : is the number of common points.

3. DATA USED FOR DERIVING THE PARAMETERS

Data used for computation of transformation of parameters broadly divided into mainland and Hatta region. Out of 6000 points in the main land total of 3778 points were common on both datums and it was satisfactorily well distributed through out the main land. Where as 92

control points were taken for Hatta region. Out of 3778 points only about 75% points were having spirit level heights and where as all the points were having the ellipsoidal heights. Like most local geodetic datums Clark1880 is basically a horizontal datum rather than three dimensional datum. In the first instant conversion of spirit level height to the ellipsoidal heights were taken place for all the points for main land and Hatta, which were on Clark1880 spheroid using the following equations:

$$\Delta h = \Delta X \cos\phi \cos\lambda + \Delta Y \cos\phi \sin\lambda + \Delta Z \sin\phi + (a \Delta f + f \Delta a) \sin^2(\phi) - \Delta a$$

Which further help to convert the grid coordinates into the geocentric coordinates. Initially the whole data has been taken for computation of the parameters. After analysis only 2966 points were selected for the main land having residual within one meter and also they represent a well distribution. For Hatta (hilly region) 88 points were selected which were having residual within 50cm and points were at all accessible places only. The points distribution are shown in DGN File-2 for main land and for the Hatta region.

The five transformation parameters associated with the Abridged Molodensky model ($\Delta X, \Delta Y, \Delta Z, \Delta a, \Delta f$) and the parameters of the Clark1880 ellipsoid. These parameters shown in Table 1

Parameter	Value	Units
ΔX	250.4177	Metres
ΔY	156.3840	Metres
ΔZ	-381.4507	Metres
Δa	-112.145	Metres
Δf	5.4751×10^{-05}	Radians
a	6377304.063	Metres
f	1/300.8017	Radians

Table 1: The Abridged Molodensky transformation parameters from WGS84 to CLARK1880

4. COMPUTATIONS

The purpose of this study is to determine the optimal transformation parameters between the local datum Clark1880 and WGS84 datum. Like most local geodetic datums, the Clark1880 spheroid is basically a horizontal datum rather than a three dimensional datum. For 75% of the stations taken for computation spirit level heights were available and for the all other stations ellipsoidal heights were available. In the absence of the spirit level heights, for the cotrol points, the orthometric heights were computed using the Dubai Precise Geoid model, to derive the ellipsoidal heights of the Clark1880 spheroid. The two basically different ways to define the transformation between WGS84 and the local Clark1880 datum are:

- To set a condition that the axes of the two ellipsoids are parallel. In this case, the transformation will be expressed with only three translations.

- Seven-parameter similarity transformation made up of three translations, three rotations and scale factor. Where the axes are not parallel and scale varies.

In this study a full seven-parameter datum transformation approach was adopted. Using Cartesian coordinates which are calculated using computed ellipsoidal heights of Clark1880 spheroid for main land and for Hatta.

4.1 Conversion of Spirit Level/Orthometric Heights into Ellipsoidal Heights

For the ellipsoidal solution, the spirit level heights (H) have to be converted into their corresponding ellipsoidal heights (h) on the Clark1880 Spheroid. The basic equation which relates the ellipsoidal height (h), the orthometric height (H) and the geoid height (N) is given by

$$h = H + N \quad (20)$$

Using the geodetic co-ordinates for the ellipsoidal solution, the orthometric heights (H) have to be converted into their corresponding ellipsoidal heights (h) on the Clark1880 Spheroid . The basic equation which relates the ellipsoidal height (h), the orthometric height (H) and the geoid height (N) is given by

$$h = H + N \quad (21)$$

Using the geodetic co-ordinates, together with the parameters of WGS84 ellipsoid ($a = 6378137m, f = 1/298.2572$), the geoid heights (N_{WGS84}) on WGS84 system above WGS84 ellipsoid at the 3778 common points were computed. The corrections (ΔN) to transform WGS84 geoid heights (N_{WGS84}) to their corresponding values (N_{clark}) in the local Clark1880 system were computed from equation (15), using the five transformation parameters ($\Delta X, \Delta Y, \Delta Z, \Delta a, \Delta f$) and the associated constants with the Abridged Molodensky model of Table 1 The Clark1880 geoid heights (N_{CLARK}) were computed from:

$$N_{CLRK} = N_{WGS84} - \Delta N \quad (22)$$

The ellipsoidal heights above Clark1880 spheroid could be computed from equation (17) as:

$$h_{clark} = H + N_{clark} \quad (23)$$

Finally, substituting equation (22) in equation (23), the ellipsoidal heights (h_{clark}) on the local Clark1881 system were computed from:

$$h_{CLARK} = H + N_{WGS84} - \Delta N \quad (24)$$

The computed ellipsoidal heights on the Clark1880 spheroid together with their Cartesian co-ordinates were finally listed.

4.2 Least Squares Estimation of the Transformation Parameters

Using the method of least squares and after some re-arrangements, Bursa-Wolf model as given by equation (17) could be re-written in a linear form as:

$$\begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & X_{CLK} & 0 & -Z_{CLK} & Y_{CLK} \\ 0 & 1 & 0 & Y_{CLK} & Z_{CLK} & 0 & -X_{CLK} \\ 0 & 0 & 1 & Z_{CLK} & -Y_{CLK} & X_{CLK} & 0 \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \Delta L \\ R_X \\ R_Y \\ R_Z \end{bmatrix} - \begin{bmatrix} X_{WGS\ 84} - X_{CLK} \\ Y_{WGS\ 84} - Y_{CLK} \\ Z_{WGS\ 84} - Z_{CLK} \end{bmatrix} \quad (25) \text{ Also the}$$

same way we can re-write Molodensky-Badekas model given by equation (18) in a linear

$$\text{form as: } \begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \mu_X & 0 & -\mu_Z & \mu_Y \\ 0 & 1 & 0 & \mu_Y & \mu_Z & 0 & -\mu_X \\ 0 & 0 & 1 & \mu_Z & -\mu_Y & 0 & \mu_X \end{bmatrix} \begin{bmatrix} dX \\ dY \\ dZ \\ \Delta L \\ R_X \\ R_Y \\ R_Z \end{bmatrix} - \begin{bmatrix} X_{WGS\ 84} - X_{CLK} \\ Y_{WGS\ 84} - Y_{CLK} \\ Z_{WGS\ 84} - Z_{CLK} \end{bmatrix} \quad (26)$$

$$\text{with: } \begin{bmatrix} \mu_X \\ \mu_Y \\ \mu_Z \end{bmatrix} = \begin{bmatrix} X_{CLK} - X_m \\ Y_{CLK} - Y_m \\ Z_{CLK} - Z_m \end{bmatrix} \quad (27)$$

The least squares observation equation for a linear mathematical model is:

$$V = A\hat{X} - L \quad (28)$$

The least squares solution for the unknown parameters (\hat{X}) together with their statistics can be computed from:

$$\hat{X} = (A^T P A)^{-1} A^T P L \quad (29)$$

$$\sum_{\hat{X}}^{\wedge} = \hat{\sigma}_0^2 (A^T P A)^{-1} \quad (30)$$

$$\hat{\sigma}_0^2 = \frac{V^T P V}{n - u} \quad (31)$$

where,

- V : the vector of residuals;
- A : the design or coefficient matrix;
- \hat{X} : the vector of unknowns;

- L : the vector of observations;
- P : the weight matrix of the observations;
- σ_0^2 : the a posterior variance of unit weight;
- n : the number of observations;
- u : the number of unknowns.

To compute various sets of transformation parameters for the main land, the two data sets of 3778 common points of mainland and in case for Hatta 92 points from the area were taken for investigation using both Bursa-Wolf and Molodensky Badekas models solution. This was used initially to filter the common points with high residual and the outlier. the common points within 1 m residual in XYZ for the main land and 50 cm residual in XYZ for the Hatta were used for further computation of parameters. A total of 2966 common points for main land and 88 common points for Hatta were filtered for computing the parameters taking an account of their distribution which are shown in DGN File-2. In order to determine various transformation parameter sets between the Clark1880 system and WGS84 system, a module from SKI Pro were used and results were compared with the program written in Fortran by Dr. Hassan Fashir. Five different sets of transformation parameters were computed using both the Bursa-Wolf and the Molodensky Badekas models. The computation residuals for 7 parameters are listed in Table 2 for main land and Table 3 for Hatta respectively. The coordinate differences between the CLARK1880 system and WGS84 system were treated as observations in the least squares solution of equation (28). Also the observations were assumed to carry equal weights. The five sets of transformation parameters are computed which consists of three shifts only ($\Delta X, \Delta Y, \Delta Z$), three shifts and scale change ($\Delta X, \Delta Y, \Delta Z, \Delta L$), three shifts, scale change and rotation about the Z-axis ($\Delta X, \Delta Y, \Delta Z, \Delta L, R_z$), three shifts and three rotations ($\Delta X, \Delta Y, \Delta Z, R_x, R_y, R_z$) and the full seven transformation parameters with scale ($\Delta X, \Delta Y, \Delta Z, \Delta L, R_x, R_y, R_z$).

4.3 Parameter Back – Substitution

In this study we set out to test the accuracy of the various transformation parameter sets. This is achieved by comparing total 3778 points in main land and 92 points in Hatta. The observed Grid coordinates (Easting Northing) of Clark1880/UTM compared with the computed coordinates by transforming the GPS derived coordinates. For transforming the coordinates the computed five set of parameters were used. By back-substitutions, Bursa-Wolf parameter in the equation (17), the observed GPS (ϕ, λ, h), coordinates were converted to Clark1880 Cartesian and further converted to Grid and compared with the observed coordinates of Clark1880. similarly Molodensky Badekas computed parameters were used and computed the coordinates as in the above case and compared with the observed ones. The accuracy achieved for the seven parameter after conversion is tabulated in Table 4 and Table 5 for main land and Hatta respectively. Out of all parameter set 7 parameters set of transformation are chosen and the statistical quantities like means, standard deviations, root square errors of the co-ordinates differences and the vector representing the total root mean square error in

X,Y,Z are computed using the standard statistical formulae. For the statistical analysis the converted data using 7 parameters have been used.

5. RESULTS AND ANALYSIS

At the moment optimal unique set of transformation parameters are not available in Dubai Emirate to transform the Coordinates and Graphics. This is mainly due to the fact that no precise Geoid information was available. Now with precise geoid model of Dubai Emirate an attempt is made to compute these parameters.

To compute transformation parameters between Clark1880 and WGS84 a set of well distributed common points on both datums has been selected. This is shown in DGN file -2 for the main land and for Hatta area. The transformation equation from geodetic to Cartesian are given in equation 1,2 and 3. From these equations it is evident that ellipsoidal heights for both systems are required. In the Clark1880 datum, only spirit levels are available hence we used Abridged Molodensky formulas (refer Equation 15) to convert reverse, the spirit level into their corresponding ellipsoidal heights using geoid model. The comparison of the Geoidal separation between the heights of these two datums are shown in the Figure 1 for the mainland and Hatta.

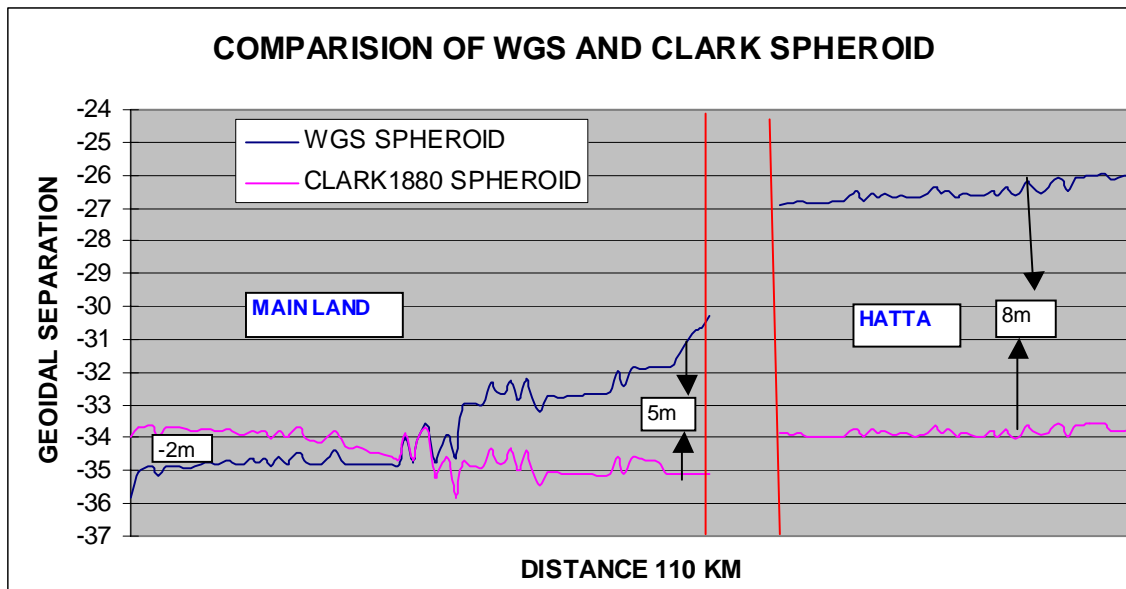


Figure 1: Comparison of WGS and CLARK spheroid

From the above figures it is evident that this separation varies between -1.8 m to 7.7m, where the separation is maximum at Hatta. The negligence of this separation will lead to inconsistent solution of parameters. From the following formulae the negligence of Geoidal separation is worked out.

$$X_{wgs} = (v + h) \cos \phi \cos \lambda ; Y_{wgs} = (v + h) \cos \phi \sin \lambda ; Z_{wgs} = (v(1 - e^2) + h) \sin \phi$$

where $h=H+N$

$$X_{clk} = (v + h) \cos \phi \cos \lambda ; Y_{clk} = (v + h) \cos \phi \sin \lambda ; Z_{clk} = (v(1 - e^2) + h) \sin \phi$$

where $h = H + 0$

$$\partial x = N \cos \phi \cos \lambda ; \partial y = N \cos \phi \sin \lambda ; \partial z = N \sin \phi$$

the inconsistency of parameter worked out at approximate position of Latitude 25° and Longitude 55° 25' in Dubai is as follows:

∂x	∂y	∂z
2.4 m	3.5 m	2.0 m

For computation of parameters main land Dubai and Hatta have taken separately, for the improvement of the accuracy of the transformation parameters. During computation of parameters for the main land a total of 2966 points were used. The 5 set of transformation parameters are computed using the Bursa-Wolf and Molodensky Badekas models. All the 5 sets with different combination of translation and rotation parameters are computed. The residuals are tabulated in Table 2. We found that both models give identical residuals but different set of parameter gives different residuals accuracy. For the SEVEN and SIX parameters all 2966 points residual of XYZ are within 1m where as for FIVE, FOUR and THREE parameters, 97%, 94% and 92% of points residuals of XYZ are within 1m respectively. For SEVEN parameter about 56% XY points residual are with in 10cm. Where as for SIX, FIVE, FOUR and THREE parameters, 47%, 9%, 20% and 16% of XY points residuals are with in 10cm. Here it is to be noted that the 5 parameters residual is the worst in this combination , it looks that the rotation about the Z axis without considering of XY rotation deteriorate the quality of the parameters. Comparison of all set of residual of parameters are drawn as curves.

After computing 5 sets of transformation parameters (3 to 7), a test was conducted to find out the transformation accuracy. A total of 3778 points were taken for testing the parameters using both Bursa-Wolf and Molodensky Badekas computed parameters. Both the models gives identical results. Out of all the set of parameters, 7 parameters look more consistent and giving good results. All 3778 points on WGS84 spheroid were taken and converted to the Clark1880 using 7 parameters on the both the models. The results were compared with the actual observed points on Clark1880. All the obtained accuracies are listed out in Table 4. Similarly all the sets of parameters are tested with the above data and results are compared. The SEVEN parameters are much superior when we compare the converted results with the observed Easting(E), Northing(N) and h (computed clark ellipsoidal height). Total of 99.1% points are within 50cm and 52% points in E,N are within 10cm. Comparing the results we have seen that SIX parameters are next better parameters where as FIVE parameters are the least better out of all. These results indicate that the scale difference and the rotation elements particularly XY are very significant and must be considered.

A study of the results in Table 4 reveals that there are no significant differences in the quantities for the conversion when using either Molodensky-Badekas or Bursa-Wolf parameters sets. Also from the statistical calculation a posterior variance of unit weight ($\hat{\sigma}_0^2$) varies from 0.09, 0.14 and 0.22 cm. for x,y and z respectively, for the full seven-parameter solution. The RMS vectors illustrate that the seven parameters set does give the best fit to the

data set than the 3,4,5 and 6 parameter sets. The precision we achieved after conversion in case of 7 parameters are superior. From the above discussion the best transformation parameters set recommended for “SEVEN PARAMETERS” of Molodensky Badekas and Bursa-Wolf . The transformation parameters from WGS84 to CLARK1880 are as follows:

MAIN LAND

7 (dX, dY,dZ,Rx,Ry,Rz & S) TRANSFORMATION PARAMETERS							
Molodensky Badekas				Bursa Wolf			
	Value	RMS	Unit		Value	RMS	Unit
Shift dX	233.45100	0.0064	m	Shift dX	114.64340	0.9711	m
Shift dY	131.59530	0.0064	m	Shift dY	79.35610	1.1391	m
Shift dZ	-395.81930	0.0064	m	Shift dZ	117.05630	1.5105	m
Rotation about RX	11.83749	0.04828	"	Rotation about RX	11.83749	0.04828	"
Rotation about RY	-11.97969	0.03559	"	Rotation about RY	-11.97969	0.03559	"
Rotation about RZ	0.97067	0.03373	"	Rotation about RZ	0.97067	0.03373	"
Scale	-18.35860	0.1352	ppm	Scale	-18.35860	0.1352	ppm
Rotation Origin (m) : Xo = 3283896.0005 ; Yo= 4749903.8592 ; Zo = 2699186.5371							

Similarly for computing the transformation parameters for the Hatta area a total of 88 common points on both the ellipsoids are used. The 5 set of transformation parameters are computed using the Bursa-Wolf and Molodensky Badekas model. All the 5 sets with different combination of translation and rotation parameters are computed. The residuals are tabulated in Table 3. We found that both models give the identical residuals but different set of parameter gives the different residuals accuracy. All 88 points residual of XYZ are within 50cm. For SEVEN parameter about 81% XY points residual are with in 10cm. Where as for SIX, FIVE, FOUR and THREE parameters, 75%, 77%, 73%and 69% of XY points residuals are with in 10cm respectively. Here it is to be noted that the 5 parameters are better than 6 parameters also it is to be noted that Bursa-Wolf parameters RMS are higher compare to other model. It looks that the rotation about the Z axis has got the significant effect on the parameters.

After computing 5 sets of transformation parameters (3 to 7), a test was conducted to find out the transformation accuracy. A total of 92 points were taken for testing the parameters using both Bursa-Wolf and Molodensky Badekas computed parameters. Both the models gives the different results. Out of all the set of parameters, 7 parameters look more consistent

and giving the good results. All 91(one point outlier) points on WGS84 spheroid were taken and converted to the Clark1880 using 7 parameters on both models. The results were compared with the actual observed points on Clark1880. All the accuracies obtained are listed in Table 5. Similarly all the sets of parameters are tested with the above data and results are compared . The SEVEN parameters are better when we compare the converted results with the observed Easting(E), Northing(N) and h (computed Clark ellipsoidal height). Total 98% points are within 50cm and 82% points in E,N are within 10cm. Comparing the results we have seen that SIX and FIVE parameters are next better parameters, also in this it has been noticed that the different models giving slightly different accuracies. In Hatta region these results indicate that the scale difference and the rotation elements is not that significant while comparing to main land Dubai.

A study of the results in Table 5 reveals that there are no significant differences in the quantities for the conversion when using either Molodensky-Badekas or Bursa-Wolf parameters sets. Also from the statistical computation a posteriori variance of unit weight ($\hat{\sigma}_0^2$) varies from 0.06, 0.13 and 0.07 cm. for x,y and z respectively, for the full seven-parameter solution. The RMS vectors illustrate that the seven parameters set does give the best fit to the data set than the 3,4,5 and 6 parameter sets. The precision we achieved after conversion in case of 7 parameters is slightly better. From the above it is recommended that the “SEVEN PARAMETERS” of Molodensky Badekas and Bursa-Wolf transformation parameters sets to be used for the Hatta area. The transformation parameters from WGS84 to CLARK1880 are as follows:

HATTA

7 (dX, dY,dZ,Rx,Ry,Rz & S) TRANSFORMATION PARAMETERS							
Molodensky Badekas				Bursa Wolf			
	Value	RMS	Unit		Value	RMS	Unit
Shift dX	229.18130	0.0085	m	Shift dX	459.70430	21.1733	m
Shift dY	124.85640	0.0085	m	Shift dY	-12.33080	25.3213	m
Shift dZ	-396.94260	0.0085	m	Shift dZ	-182.51510	38.3242	m
Rotation about RX	4.74440	1.20432	"	Rotation about RX	4.74440	1.20432	"
Rotation about RY	-3.89482	0.85963	"	Rotation about RY	-3.89482	0.85963	"
Rotation about RZ	-9.80798	0.69953	"	Rotation about RZ	-9.80798	0.69953	"
Scale	-16.10110	2.9933	ppm	Scale	-16.10110	2.9933	ppm
Rotation Origion (m) : Xo = 3227836.4629 ; Yo= 4810942.4290 ; Zo = 2659452.6949							

6. CONCLUSION AND RECOMMENDATIONS

The optimal transformation parameters sets computed for main land and Hatta of Dubai Emirate between WGS84 datum and CLARK1880 datum. The full seven parameters set is recommended for the transformation using both the Molodensky Badekas and Bursa-Wolf model. The parameters were estimated by using 2966 points which are sufficiently well distributed in the main land including desert and 88 points are distributed wherever access are available in Hatta. The data conversion using the above transformation parameters are tested against 3778 points in the main land and 92 points in Hatta area. The availability of the

precise Geoid model made it possible to compute a unique set of transformation parameters for both mainland Dubai and Hatta. For any WGS84 ellipsoidal points a precision of the transformation is of the order of 1-10 cm in mainland and Hatta. where as for any CLARK1880 points conversion precision between 10 -50 cm . These parameters could be used both inland and off-shore.

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APPENDICES

Transformation computation Residuals table for 2966 common points

7 PARAMETERS (dX, dY, dZ, Rx, Ry, Rz & S)

TABLE-2

Computation residuals for 7 PARAMETERS										Remarks
POINTS	MOLODENSKEY BADEKAR				BOURSA WOLF					
Within	XY	%	XYZ	%	XY	%	XYZ	%		
10 cm	1652	56	1241	42	1652	56	1241	42	all points within 1 m	
15 cm	2415	81	1993	67	2415	81	1993	67		
20 cm	2739	92	2516	85	2739	92	2516	85		
25 cm	2912	98	2819	95	2912	98	2819	95		
50 cm	2959	100	2940	99	2959	100	2940	99		
1 cm	2966	100	2966	100	2966	100	2966	100		

Transformation computation Residuals table for 88 common points at Hatta

7 PARAMETERS (dX, dY, dZ, Rx, Ry, Rz & S)

TABLE 3

Computation residuals for 7 PARAMETERS										Remarks
POINTS	MOLODENSKEY BADEKAR				BOURSA WOLF					
Within	XY	%	XYZ	%	XY	%	XYZ	%		
10 cm	71	81	70	80	71	81	66	75	all points within 50cm	
15 cm	79	90	76	86	79	90	74	84		
20 cm	83	94	82	93	83	94	82	93		
25 cm	83	94	82	93	83	94	82	93		
50 cm	88	100	88	100	88	100	88	100		

Accuracy after transformation using the Transformation Parameters

7 PARAMETERS (dX, dY, dZ, Rx, Ry, Rz & S)

TABLE-4

CONVERSION OF 3778 POINTS USING 7 PARAMETER FROM WGS84 TO CLARK1880										Remarks
POINTS	MOLODENSKEY BADEKAR				BOURSA WOLF					
Within	XY	%	XYZ	%	XY	%	XYZ	%		
10 cm	1943	52	1654	44	1944	52	1654	44	34 points outlier	
15 cm	2751	73	2527	67	2752	74	2527	67		
20 cm	3150	84	3006	80	3157	84	3006	80		
25 cm	3443	92	3317	89	3451	92	3317	89		
50 cm	3675	98	3676	98	3683	98	3676	98		
1 cm	3744	100	3744	100	3744	100	3744	100		

Accuracy after transformation using the Transformation Parameters at Hatta

7 PARAMETERS (dX, dY, dZ, Rx, Ry, Rz & S)

TABLE -5

CONVERSION OF 92 POINTS USING 7 PARAMETER FROM WGS84 TO CLARK1880										Remarks
POINTS	MOLODENSKEY BADEKAR				BOURSA WOLF					
Within	XY	%	XYZ	%	XY	%	XYZ	%		
10 cm	75	82	70	77	75	82	70	77	1 point outlier	
15 cm	82	90	80	88	82	90	80	88		
20 cm	85	93	84	92	85	93	84	92		
25 cm	85	93	84	92	85	93	84	92		
50 cm	89	98	89	98	89	98	89	98		
1 cm	91	100	91	100	91	100	91	100		

